

On the Fractional-Order Derivative Effects on the ABS Robust Control Performance

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ABSTRACT This study conducts a comparative analysis of sliding mode control (SMC) and fractional-order sliding mode control (FOSMC) for application in antilock braking systems (ABS). Based on foundational principles of theoretical mechanics, the ABS dynamics are modeled as a single-input system to analyze wheel-slip regulation under diverse and variable road conditions. Both the conventional SMC and the proposed FOSMC are designed using a Lyapunov-based approach to ensure robust stability, with the latter incorporating fractional-order derivatives to refine the sliding surface and dynamic response. The conventional SMC method, while demonstrating strong robustness and disturbance rejection capabilities, is found to induce persistent chattering during transient phases, which can compromise system reliability and actuator longevity. By contrast, the FOSMC controller enhances transient behavior by attenuating chattering and yielding smoother, more consistent wheel-slip tracking. The inclusion of fractional-order terms contributes to faster convergence and improved adaptation to abrupt changes in road friction, though it introduces increased computational complexity. Numerical simulations validate the performance of both controllers across multiple driving scenarios, including dry, wet, and icy road conditions. Results confirm that FOSMC significantly reduces chattering, accelerates system convergence, and maintains stable braking performance with greater consistency compared to conventional SMC, establishing its potential for implementation in advanced ABS designs.

KEYWORDS

Antilock braking system
Sliding mode control
Fractional order control
Robust control

INTRODUCTION

Since the introduction of the automobile, the importance of safety in trajectory control, particularly regarding braking systems, has been widely recognized. This has led to the development of innovative safety systems capable of preventing wheel lock during sudden or heavy braking, thereby maintaining tire traction and steering ability, such as antilock braking systems (ABS) (Pretagostini *et al.* 2020). Depending on the types of actuators used, there are three braking modes: hydraulic friction braking (HFB), hybrid braking, and electric motor braking (EMB). However, hydraulic friction braking is the most commonly employed method due to its efficiency and reliability (Pretagostini *et al.* 2020). ABS technology is specifically designed to adjust braking pressure, ensuring wheel slip remains within a range of 10% to 20% (Gowda and Ramachandra 2017). This optimization maximizes braking force while maintaining the lateral steering force necessary for directional control. To design effective ABS controllers, comprehending vehicle dynamics and wheel slip behavior is essential (Jazar 2008;

Geleta *et al.* 2023).

During braking, the interaction between a vehicle's wheels and the road surface produces tractive forces, which can be quantified as the product of the road coefficient and vertical forces. This coefficient can be expressed as a function of wheel slip. In ABS, the challenge lies in setting a desired wheel slip ratio that reflects the difference between wheel velocity and road velocity. Therefore, the primary objective of ABS design is to maintain the slip ratio as close as possible to the optimal value of 0.2, where tractive force is maximized (Chen *et al.* 2022). This condition is critical for effective braking, as achieving a slip ratio of zero is not feasible. ABS represents a significant advancement in automotive active safety, aimed at preserving vehicle stability and maneuverability during emergency braking by preventing wheel lock. The major challenge arises from the inherent nonlinearity and uncertainties in the vehicle-tire-road dynamic model. The design of ABS control faces substantial uncertainties related to variations in road grip, vehicle load, and tire properties (Chen *et al.* 2022). To tackle these challenges, extensive research has been directed toward optimizing wheel slip control, a key parameter for enhancing system efficiency and robustness (Zhao *et al.* 2024).

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Classical controllers, such as bang-bang and PID controllers, have been proposed and calibrated for specific road conditions but often exhibit inefficiency and instability during braking (Nemah 2018). Various studies in braking control have explored sliding mode control (SMC) methods (Bi *et al.* 2024). For instance, researchers have proposed control strategies for ABS based on vehicle longitudinal dynamics (Schinkel and Hunt 2002). However, limitations in these nonlinear models sometimes fail to capture the intricate dynamics of real-world vehicle behavior on diverse surfaces. Consequently, the performance of proposed controllers can vary based on specific vehicle parameters and road configurations, and chattering remains a significant issue in SMC implementations. Several researchers have focused on minimizing chattering, a prominent feature of SMC systems. For example, algorithms for ABS have been developed comparing the Lyapunov-based sliding mode controller (LSMC) and the reaching-law-based sliding mode controller (RSMC) (Chereji *et al.* 2021), which show robust performance in managing uncertainties with fewer tuning parameters. Despite their promise, limitations persist. The models employed are often simplified and may not fully account for the complexities of real-world conditions, where uncertainties can fluctuate significantly. While these algorithms aim to streamline computational complexity, their effectiveness can still be sensitive to parameter adjustments, and they may not adequately address vibration issues that could affect user comfort and system longevity. Moreover, most validations have taken place in controlled settings, which may not accurately reflect real driving situations, raising questions about the scalability of these controllers in more complex systems.

To address the nonlinearities and uncertainties inherent in Anti-lock Braking System (ABS) frameworks, along with the challenges posed by chattering in sliding mode control, a robust control strategy has been developed to enhance the braking system's resilience against disturbances (Garcia Torres *et al.* 2022; Khadr *et al.* 2024). Recent advancements have introduced methods that leverage fuzzy logic to estimate the road adhesion coefficient, facilitating real-time optimization of slip rates (Gengxin *et al.* 2022). Evaluating these methods against traditional control strategies, such as PID, has yielded significant improvements in braking distance and response times, ultimately enhancing safety (Latreche and Benagoune 2015). Nonetheless, the simplified models employed may inadequately capture the wide spectrum of road conditions, and the accuracy of adhesion coefficient estimations can be compromised by unmodeled uncertainties. Additionally, the sensitivity to parameter variations and potential chattering issues within sliding mode controllers can negatively impact overall performance. Recently, Sina *et al.*, proposed an innovative control system that incorporates fuzzy logic control (FLC) to effectively mitigate issues related to nonlinearity, uncertainties, and disturbances, without the need for human intervention during braking. They introduced an adaptive fuzzy sliding mode longitudinal control strategy tailored for vehicles with ABS, aimed at minimizing braking distances (Namaghi and Moavenian 2019; Abdul Zahra and Abdalla 2020; Max *et al.* 2021). Concurrently, neural network control (NNC) has emerged as a complementary approach, with work by Sebanovic *et al.* developing artificial neural networks designed for real-world data-driven virtual sensors in vehicle suspension systems. Their research includes applications such as neural network-based model reference control for electric vehicle braking and multi-task learning driven by deep graph neural networks (Sabanovic *et al.* 2024; Vodovozov *et al.* 2021; Xiao 2022). While these methodologies exhibit promising performance, their design and implementation can be complex and resource-intensive. Researchers continue to refine

control designs to bolster braking controller robustness through established mathematical frameworks. Researchers have designed controllers that regulate wheel slip to desired values by combining sliding mode control with fractional calculus (Tang *et al.* 2013). It is of great importance to recall that many works presented in literature consider the integer ABS which make the performance of the controllers questionable in real time. With the ABS fractional order consideration, the dynamic models used for ABS may oversimplify the intricate variables affecting vehicle behavior during braking, which can impede the adaptive controller's ability to accurately estimate uncertainties encountered in real-world conditions, potentially degrading system performance. Additionally, vibration issues commonly linked to sliding mode controllers may not be fully resolved, adversely affecting user comfort and system durability. The fractional ABS aim to maintain robustness against external disturbances, featuring a dynamic fractional order sliding surface that enhances both response speed and controller flexibility.

This study systematically investigates two primary control methodologies: classic sliding mode control (CSMC) and fractional order sliding mode control (FOSMC). Comparing these methods is crucial for illuminating their respective performances and advantages. A clear presentation of these comparisons is necessary, supported by a concise table detailing the performance metrics of each method. Importantly, fractional order modeling in ABS offers enhanced control flexibility compared to its integer-order counterparts, making it a central focus of this research. To address existing gaps in the literature, this study carefully compares CSMC and FOSMC, specifically underscoring slip ratio regulation in ABS. Among the benefits of FOSMC is its advanced modeling capability, which facilitates better adaptability to parametric variations and external disturbances. The integration of fractional-order differential operators improves the dynamics of the sliding surface, resulting in faster convergence and greater robustness of the controller. Despite these advantages, comprehensive comparisons of CSMC and FOSMC within the ABS framework, accounting for realistic disturbances such as variations in road adhesion and sensor noise, are still limited. Consequently, this research aims to quantify the contributions of fractional order modeling in terms of tuning flexibility and chattering reduction, while also evaluating performance enhancements in slip regulation, stopping distance, and convergence time in comparison to classical benchmarks like integer-order sliding mode control.

This paper focuses primarily on constant actuator disturbances and modelling uncertainties, highlighting their impact on vehicle longitudinal dynamics, particularly during braking. It proposes a robust controller based on a fractional-order sliding mode to address these challenges compared to the conventional sliding mode. Among the disturbances mentioned are variations in vehicle mass, wheel radius and wheel inertia, which can influence the system. The design presented aims to manage the longitudinal dynamics of vehicles in the context of ABS system. We take into account constant actuator disturbances and modelling uncertainties, which vary depending on road conditions. These disturbances, such as brake pad wear, can cause a lag between the control output and the actuator response, compromising braking performance. In addition, variations in vehicle parameters and road surface conditions can affect the coefficient of friction and, consequently, stability during braking. In response to these challenges, the paper proposes a robust controller capable of maintaining the desired slip trajectory despite uncertainties. Using Lyapunov theory, the controller ensures system stability in a dynamic setting, taking into

account various scenarios, including actuator failures. Simulations show that the fractional-order controller outperforms traditional controllers, ensuring accurate and rapid tracking of reference values under normal conditions as well as in the presence of model variations or failures. This approach highlights the importance of robust design for active vehicle safety systems.

To bridge this gap, this paper presents a comparative study between classic sliding mode control and fractional order sliding mode control for robust controller design in anti-lock braking systems. The principal contributions of this work are as follows:

- Design of classical and fractional order sliding mode control laws using a quarter-vehicle model that incorporates fractional-order dynamics.
- A rigorous stability analysis base on Lyapunov function adapted to fractional-order systems.
- Comprehensive numerical validation across diverse operating conditions (dry asphalt, concrete, snow, and ice), with evaluation of performance indices such as ITAE and ITE.
- Presentation of a performance summary table highlighting improvements offered by the fractional-order approach, especially in the presence of unmodeled disturbances.

The remainder of this paper is organized as follows: Section 2 details the classic sliding mode control using a quarter-vehicle model and problem formulation. Section 3 presents the design and stability proof of the proposed controller. Section 4 discusses the simulation results and performance analysis, and Section 5 provides concluding remarks.

CLASSIC SLIDING MODE CONTROL FOR ABS

The dynamics of the antilock braking system (ABS) can be defined as the collection of behaviors and responses of a vehicle's braking system, influenced by various parameters such as vehicle speed, tire grip on the road, and the forces applied during braking, Fig. 1.

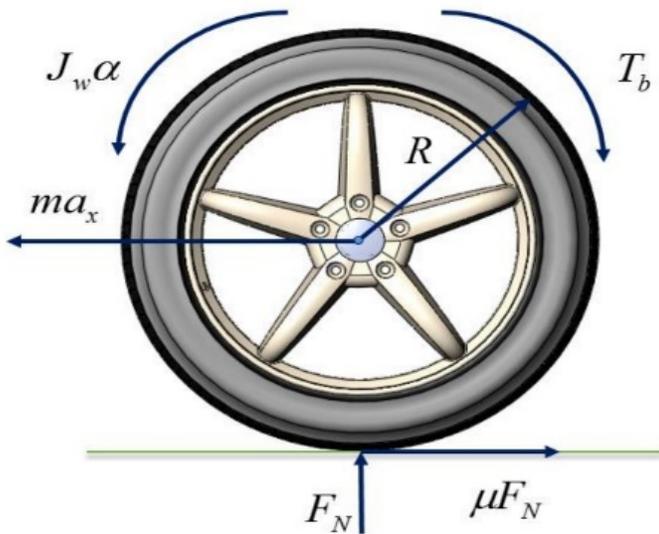


Figure 1 The antilock braking system model diagram

The vehicle wheel is equipped with a disk braking system. The diagram below illustrates the dynamics of a wheel, showing key components such as torque T_b , angular acceleration, moment of

inertia J_w , linear acceleration a_x , normal force F_N , and frictional force $\mu(F_N)$ (Abdul Zahra and Abdalla 2020). The torque applied to the wheel induces angular acceleration, while the moment of inertia determines the resistance to changes in rotation. The linear acceleration reflects the wheel's forward motion, influenced by the frictional force that enables grip on the road. These interactions are governed by Newton's laws (Garcia Torres et al. 2022; Abdul Zahra and Abdalla 2020). Understanding these dynamics is crucial for systems like anti-lock braking systems (ABS), ensuring vehicle stability and preventing wheel lock-up during braking. The normal force $F_z = mg$ refers to the perpendicular force exerted by the road surface on the wheels of a vehicle, Fig. 1. This force is crucial as it directly influences tire grip and braking performance, with m the mass of the vehicle. F_x is the road longitudinal friction force, which can be given by the coulomb law ;

$$F_x = \mu(\lambda)F_N \quad (1)$$

The road coefficient of adhesion μ , depends on many factors including tire-road condition. The wheel velocity and the value of wheel slip λ is defined as follow:

$$\lambda = \frac{v_x - r_w}{v_x} \quad (2)$$

with

$$\dot{\lambda} = -((1\lambda)/(mv_x) + r^2/(Jv_x))\mu F_N + r/(Jv_x)T_B \quad (3)$$

The wheel slip lies in $[0,1]$. If $\lambda = 0$, it indicates that the wheel and vehicle velocities are the same, if $\lambda = 1$, it indicates that the wheel is locked up. Table 1 shows the friction model parameters for various road surfaces.

Table 1 Friction model parameters for different road conditions

Surface_conditions	C1	C2	C3
Dry asphalt	1.2801	23.99	0.52
Wet asphalt	0.857	33.822	0.347
Dry concrete	1.1973	25.168	0.5373
Snow	0.1946	94.129	0.0646
Ice	0.05	306.39	0.000

The tire friction model is a nonlinear static function of several physical variables such as wheel slip and vehicle velocity, Fig. 2. In this paper, the tire friction model introduced by Burckhardt is used. The model is as follows:

$$\mu(\lambda, v_x) = C_1(1 - e^{-C_2\lambda}) - C_3\lambda \quad (4)$$

Where C_1 is the maximum value of friction curve, C_2 is the friction curve shape and C_3 is the friction curve difference between the maximum value and the value corresponding to $\lambda = 1$.

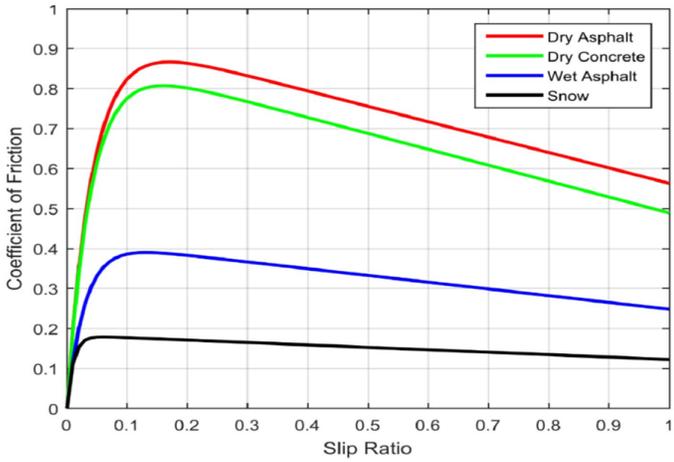


Figure 2 Coefficients of road friction versus wheel slip ratio

System formulation

In this section, a classic sliding mode controller is proposed for the ABS. The control objective is to find a control law so that the slip can track the desired trajectory λ_d . Define the tracking error as follows:

$$e = \lambda_d - \lambda \quad (5)$$

The first step is the choice of a suitable sliding surface Eq. (4).

$$s = \dot{e} + k_1 e \quad (6)$$

where k_1 denotes a positive constant, gain of robustness. From the Eq. (4) we define the time derivation of s ;

$$\dot{s} = \ddot{e} + k_1 \dot{e} \quad (7)$$

The new fast terminal dynamic sliding surface is chosen as:

$$\zeta = s + k_2 s + k_3 \int s^{(p/q)} dt \quad (8)$$

Where k_2 , p and q are positive constants and $p > q \in \mathbb{Z}$. By substituting Eqs. (3) and (5) into Eq. (8) yields:

$$\begin{aligned} \dot{\zeta} = & -\dot{\lambda} + k_2 \left[-\ddot{\lambda} - k_1 \left(\frac{1-\lambda}{mv_x} + \frac{r^2}{Jv_x} \right) \right. \\ & \left. \mu F_N - k_1 \frac{T_b R}{Jv_x} + k_3 (-\dot{\lambda} - k_1 e)^{p/q} \right] \end{aligned} \quad (9)$$

Considering the constraint $\dot{\zeta} = 0$, torque of the control can be obtained as follows:

$$\begin{aligned} T_b = & \frac{Jv_x}{r} \left[\left(\frac{\mu F_N - \dot{\lambda}}{mv_x} + \frac{\mu F_N r}{Jv_x} \right) - \frac{\ddot{\lambda}}{k_1 k_2} \right. \\ & \left. - \frac{\dot{\lambda}}{k_1} + \frac{k_3}{k_1 k_2} (-\dot{\lambda} - k_1 e)^{p/q} + k_{11} \text{sgn}(\zeta) \right] \end{aligned} \quad (10)$$

where the switch gain k_{11} is a positive, and

$$\text{sgn}(\zeta) = \begin{cases} +1, & \zeta > 0 \\ 0 & \zeta = 0 \\ -1 & \zeta < 0 \end{cases} \quad (11)$$

Substituting Eq. (9) into (10) results in:

$$\dot{\zeta} = -2\ddot{\lambda} - \dot{\lambda} - k_1 k_2 k_{11} \text{sgn}(\zeta) \quad (12)$$

It should be noted that the acceleration parameters $a_p = -2\ddot{\lambda} - k_2 \dot{\lambda}$ compensates the controller torque T_b . Therefore the Eq. (12) can be expressed as follows:

$$\dot{\zeta} = -k_1 k_2 k_{11} \text{sgn}(\zeta) \quad (13)$$

Theorem 1: If the control law (10) is applied to the wheel slip dynamics (8), the wheel slip trajectory will converge to the newly defined surface in (12). Consequently, the traction error (7) will diminish to zero in a finite time.

Proof: Considering the candidate Lyapunov function,

$$V = |\zeta| \quad (14)$$

The derivative of Eq. (14) leads to

$$\dot{V} = \dot{\zeta} \zeta \quad (15)$$

Therefore,

$$\dot{V} = \dot{\zeta} \text{sgn}(\zeta) \quad (16)$$

Substitution of Eq. (13) into Eq. (16) results in

$$\dot{V} = -k_1 k_2 k_{11} \text{sgn}(\zeta)^2 \leq 0 \quad (17)$$

The tracking error decays to zero if the SMC parameters and k_1, k_2, k_3 and k_{11} are chosen appropriately. Eq. (30) can be rewritten as:

$$\dot{V} = \frac{d|\zeta|}{dt} - k_1 k_2 k_{11} \quad (18)$$

From Eq. (18) we have the finite time of convergence;

$$dt = \frac{d|\zeta|}{-k_1 k_2 k_{11}} \quad (19)$$

Taking integral of both sides of Eq. (19) from $t=0$ to t_f ;

$$\int_0^{t_f} dt = \int_{|\zeta(0)|}^{|\zeta(t_f)|} \frac{d|\zeta|}{-k_1 k_2 k_{11}} \quad (20)$$

By setting $\zeta(t_f) = 0$ and the finite time of convergence can be demonstrated as follows:

$$t_f = \frac{|\zeta|}{-k_1 k_2 k_{11}} \Big|_{|\zeta(0)|}^{|\zeta(t_f)|} = \frac{|\zeta(0)|}{k_1 k_2 k_{11}} \quad (21)$$

Consequently, it can be concluded that the system successfully tracks the desired reference trajectory within a finite time. Thus, the proof is now complete.

System formulation with disturbance

According to the nonlinearity that characterizes the ABS, we design an adaptive sliding mode controller (SMC) that takes into account uncertainties, including constant actuator disturbances and modeling uncertainties, as well as variations in road conditions. Consequently, the dynamics of the system can be expressed as:

$$\dot{\lambda} = - \left(\frac{1-\lambda}{mv_x} + \frac{r^2}{Jv_x} \right) \mu F_N + \frac{r}{Jv_x} T_B + \zeta_d \quad (22)$$

Where ζ_d represents the lumped model uncertainties and external disturbances. In the design process, it was assumed that the upper bound of the lumped uncertainty is known. However, in practical applications, determining this bound can be challenging.

Therefore, an adaptive control law is incorporated into the proposed controller to adjust the value of the upper bound of the lumped uncertainty ζ_d .

Replacing k_{11} by k_{33} in Eq. (10), adaptive finite time dynamic sliding mode control law is derived as follows:

$$T_b = \frac{Jv_x}{r} \left[\left(\frac{\mu F_N - \lambda}{mv_x} + \frac{\mu F_N r}{Jv_x} \right) - \frac{\ddot{\lambda}}{k_1 k_2} - \frac{\dot{\lambda}}{k_1} + \frac{k_3}{k_1 k_2} \right. \\ \left. (-\dot{\lambda} - k_1 e)^{p/q} + k_{33} \text{sgn}(\zeta) \right] \quad (23)$$

We define the new surface

$$\dot{\zeta} = \ddot{s} + k_2 \dot{s} + k_3 s^{p/q} \quad (24)$$

$$\dot{\zeta} = -\ddot{\lambda} + k_2 \left[-\dot{\lambda} - k_1 \left(\frac{1-\lambda}{mv_x} + \frac{r^2}{Jv_x} \right) \mu F_N - k_1 \frac{T_b R}{Jv_x} + \zeta_d \right] + k_3 (-\dot{\lambda} - k_1 e)^{p/q} \quad (25)$$

By developing Eq. (25) has been simplify as follows:

$$\dot{\zeta} = -2\ddot{\lambda} - 2\dot{\lambda}k_2 - k_1 k_2 k_{33} \text{sgn}(\zeta) + k_1 k_2 \zeta_d \quad (26)$$

The derivative of the new surface is defined as follows:

$$\dot{\zeta} = k_1 k_2 (\zeta_d - k_{33} \text{sgn}(\zeta)) \quad (27)$$

To determine the parameter of robustness, let us consider the candidate Lyapunov function as follows;

$$V = \frac{1}{2} \zeta^2 + \frac{1}{2\eta} \tilde{k}_{33}^2 \quad (28)$$

where k_{33} is the estimate of k_{11} , η is a non-zero positive constant and $\tilde{k}_{33} = k_{11} - k_{33}$. The derivative defined as;

$$\dot{V}_\zeta = \dot{\zeta} \zeta + \frac{1}{\eta} \dot{\tilde{k}}_{33} \tilde{k}_{33} \quad (29)$$

Considering the lumped disturbance ζ_d and noting that $k_{11} - |\zeta_d| = \eta > 0$

$$\dot{V}_\zeta = k_1 k_2 \zeta (\zeta_d - k_{33} \text{sgn}(\zeta)) + \frac{1}{\eta} \dot{\tilde{k}}_{33} \tilde{k}_{33} \\ = k_1 k_2 \zeta (\zeta_d - k_{11} \text{sgn}(\zeta) + \tilde{k}_{33} \text{sgn}(\zeta)) + \frac{1}{\eta} \dot{\tilde{k}}_{33} \tilde{k}_{33} \\ = k_1 k_2 \zeta (\zeta_d - k_{11} \text{sgn}(\zeta) + \tilde{k}_{33} \text{sgn}(\zeta)) + \frac{1}{\eta} \dot{\tilde{k}}_{33} \tilde{k}_{33} \\ = k_1 k_2 (-|\zeta| \theta) + \tilde{k}_{33} \left(k_1 k_2 |\zeta| + \frac{1}{\eta} \dot{\tilde{k}}_{33} \right) \quad (30)$$

Knowing that $\dot{\tilde{k}}_{33} = 0 - \dot{k}_{33}$ and to make $\dot{V} - \dot{\zeta} > 0$, the adaptive law is designed as follows;

$$\dot{k}_{33} = k_1 k_2 \eta |\zeta_d| \quad (31)$$

FRACTIONAL DERIVATIVE SLIDING MODE CONTROLLER FOR ANTILOCK BRAKING SYSTEM

These dynamics can be modeled using fractional order derivatives to describe more complex and nonlinear behaviors. The incorporation of fractional derivatives allows for a better representation of memory effects and richer dynamics that cannot be fully captured by integer-order models. The fractional derivative is a mathematical concept that generalizes the notion of the classical derivative to non-integer orders. The continuous integro-differential operated as;

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha}, & \alpha > 0, \\ 1, & \alpha \\ \int_a^t (d\tau)^{-\alpha}, & \alpha < 0, \end{cases} \quad (32)$$

Where a and t are the lower and upper limits and $\alpha (\alpha \in R)$ is the order of the operation.

Let's consider the anti-lock braking systems equations defined as

$$\frac{d^\alpha v}{dt^\alpha} = -\mu(\lambda)g \\ \frac{d^\alpha \omega}{dt^\alpha} = \frac{r\mu(\lambda)}{J} mg - \frac{1}{J} T_b \\ \frac{d^\alpha \lambda}{dt^\alpha} = -\left(\frac{1-\lambda}{mv} + \frac{r^2}{Jv} \right) \mu(\lambda)mg + \frac{r}{Jv} T_b \quad (33)$$

Our aim is to control the ABS

Constructing a true model of the dynamics of ABS is the first step in the procedure of the controller design. Practically, the vehicle dynamics is very complex, it is impracticable to consider all relevant characteristics of the vehicle when designing controller. In this paper, a simple but effective quarter-car model is adopted, Fig. 2. This model is obtained from a straight line braking on flat road. In quarter-vehicle model, the lateral and vertical motions and the interaction between the four wheels of the vehicle are neglected. Applying Newton's second law to the vehicle and wheel, respectively, one can obtain the dynamic equations of the vehicle and wheel as;

$$m \frac{d^\alpha v_x}{dt^\alpha} = -F_x \quad (34)$$

And

$$J \frac{d^\alpha \omega_r}{dt^\alpha} = -T_b + rF_x \quad (35)$$

With the actual slip;

$$\frac{d^\alpha \lambda}{dt^\alpha} = -\left(\frac{1-\lambda}{mv} + \frac{r^2}{Jv} \right) \mu(\lambda)mg + \frac{r}{Jv} T_b \quad (36)$$

with

$$D^\alpha \lambda = \lambda_d - \lambda \quad (37)$$

In practice, the dynamics of the ABS represents more nonlinearities. To enhance precision, it is essential to account for model uncertainties and lumped disturbances. Consequently, we focus on Eq. (45),

$$\frac{d^\alpha \lambda}{dt^\alpha} = -\left(\frac{1-\lambda}{mv_x} + \frac{r^2}{Jv_x} \right) \mu F_N + \frac{r}{Jv_x} (T_b + c_a) + \zeta_d \quad (38)$$

Where ζ_d represents the cumulative effect of uncertainties and disturbances, and c_a denotes a constant actuator fault. The following assumption will be maintained throughout Eq. (39).

$$d \leq p|\zeta|^{0.5} \quad (39)$$

We define the surface as follows;

$$\zeta_\lambda = D^\alpha e + k_\alpha e \quad (40)$$

Considering Eq. (47) the new surface follows;

$$\dot{\zeta}_\lambda = D^{\alpha+1} e(t) + k_\alpha \dot{e} \quad (41)$$

Assuming the sliding mode controller comprises two types of controllers: an equivalent controller and a discontinuous controller, Eq. (42);

$$u(t) = T_b(t) = U_{eq} + U_n(t) \quad (42)$$

By substituting Eq. (38) with Eq. (41), the equivalent controller is giving as follows;

$$U_{eq} = \frac{Jv_x}{r} \left[D^{\alpha+1} \lambda_d + \left(\frac{1-\lambda}{v_x m} - \frac{r^2}{Jv_x} \right) \mu F_N + k_\alpha e(t) \right] \quad (43)$$

Considering the discontinuous controller $U_n(t) = K \text{sign}(\zeta_\lambda)$, we have;

$$T_b(t) = \frac{Jv_x}{r} \left[D^{\alpha+1} \lambda_d + \left(\frac{1-\lambda}{v_x m} - \frac{r^2}{Jv_x} \right) \mu F_N + k_\alpha e(t) \right] + K \text{sgn}(s_\lambda) \quad (44)$$

Considering the nonlinear state model including uncertainties and disturbances also the actuator fault in Eq. (38) which controlled by the control law given in Eq. (44), the dynamic wheel slip will coincide with the selected surface Eq. (40). Thus proof by the classical Lyapunov function:

$$V_L = \frac{1}{2} \zeta_\lambda^2 \quad (45)$$

Taking time derivative of (51) results:

$$\dot{V}_L = \zeta_\lambda \dot{\zeta}_\lambda < 0 \quad (46)$$

By developing, the next equation is derived from the previous one:

$$\dot{V}_L = -\zeta_\lambda \left[D^\alpha \lambda_d \left(1 - \frac{r}{Jv_x} \right) + \frac{r}{Jv_x} K \text{sgn}(\zeta_\lambda) \right] \quad (47)$$

Where $g = D^\alpha \lambda_d (1 - r/(Jv_x))$ is positive, after simplification, we have:

$$\dot{V}_L = -\zeta_\lambda \left(g + \frac{r}{Jv_x} K |\zeta_\lambda| \right) < 0 \quad (48)$$

Eq.48 is positive, and its time derivative in the above equation is negative. According to Lyapunov's theorem, this ensures the stability of the system.

SIMULATION RESULTS

The results section aims to evaluate and compare the braking response and behaviour of a vehicle using two different control strategies: a classical first-order sliding mode controller and a high-order ($n > 2$) fractional derivative sliding mode controller, which incorporates a super-twisting approach to improve robustness, with finite time. Simulations were conducted using MATLAB/Simulink, with all tests performed on dry asphalt surfaces while accounting for uncertainties and disturbances. To reduce chattering effects, the control scheme employed a saturation function, as defined in (59), instead of the traditional sign function.

$$\text{sat} \left(\frac{\zeta}{\phi} \right) = \begin{cases} \text{sgn} \left(\frac{\zeta}{\phi} \right), & \left| \frac{\zeta}{\phi} \right| \geq 1 \\ \frac{\zeta}{\phi}, & \left| \frac{\zeta}{\phi} \right| < 1 \end{cases} \quad (49)$$

Table 2 The main parameters of the ABS systems are selected as follows

Description	Value	Units
Gross vehicle weight	342	kg
Radius of tire	0.31	m
Torque	2000	Nm
Moment of inertia of wheel	1.33	kg/m ²
Gravity	9.81	m/s ²
Speed	20	m/s

Scenario 1: The classical first-order sliding mode controller

The results in this scenario were obtained with the following SMC parameters (integer order, relative degree 1):

The linear gain $c=20$, robustness gain $k=20$, boundary layer thickness $\varphi = 0.02$, maximum torque of 2000 Nm, $\alpha_{val} = 0.3$ and Eq.

By considering the command torque $T(bcmd) = \frac{IV}{R} [\varphi_{term} - ce - k \text{sat}(e, \varphi)]$ and $\varphi_{term} = m\mu g \left[\frac{R^2 m}{J} + \frac{(1-\lambda)}{V} \right]$ to simplify the numerical simulation, where $e = \lambda_d - \lambda$.

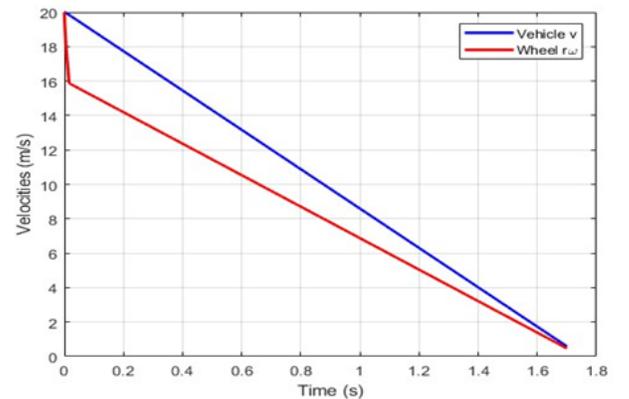


Figure 3 Wheel and vehicle velocities with classic SMC

Fig. 3. elucidates the dynamics of wheel and vehicle speeds regulated by the conventional sliding mode control strategy. This approach yields a linear and synchronous decrement in both speeds, culminating in the complete cessation of vehicle motion over a braking period of 1.7 seconds. Such a control methodology effectively manages the deceleration profile, ensuring that both the wheel and overall vehicle velocities decline in a coordinated manner. The linear reduction in speed is indicative of a systematic braking strategy that optimizes the transition from motion to rest, which is critical for maintaining vehicle stability and driver safety. Furthermore, the synchronous behavior of the wheel and vehicle speeds reinforces the efficacy of the sliding mode control in maintaining desired performance specifications, particularly under varying operational conditions.

Overall, this analysis underscores the importance of control strategies in automotive systems, particularly in enhancing safety and optimizing braking performance during critical maneuvers. The linear approach adopted here not only fulfills the technical requirements of braking but also aligns with broader principles of vehicle dynamics, reinforcing the intricate relationship between control theory and practical applications in automotive engineering.

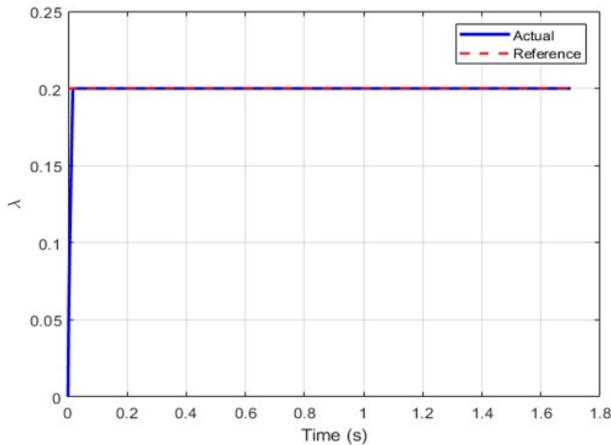


Figure 4 Longitudinal slip with classic SMC.

Fig. 4. depicts the longitudinal slip behavior managed by classic sliding mode control. This approach is predicated on the assumption that the system remains highly faithful to the desired reference slip $\lambda = 0.2$, ensuring optimal performance during braking scenarios. In this model, the convergence time to the reference slip is negligible, indicating that the controller rapidly adjusts to variations, effectively maintaining the target slip values. This swift response is critical in enhancing the vehicle's stability and control during braking, particularly in emergency situations where quick adjustments are necessary.

As illustrated in Fig. 4, the vehicle's stopping process occurs at precisely $t = 1.7$ seconds. This timing highlights the efficacy of the control mechanism in achieving the desired outcome without significant delays. By adhering closely to the reference slip, the system helps prevent wheel lock-up, thus maintaining traction and control. The classic sliding mode control method effectively balances the need for rapid response and stability, ensuring that the vehicle behaves predictably under various braking conditions. The results underscore the importance of this control strategy in modern Anti-lock Braking Systems (ABS), facilitating improved

safety and performance. Overall, the behavior exhibited in the figure exemplifies the effectiveness of classic sliding mode control in controlling longitudinal slip, further supporting the robustness of ABS systems in real-world applications. Continuous refinements in this control approach may lead to even greater advancements in braking technology.

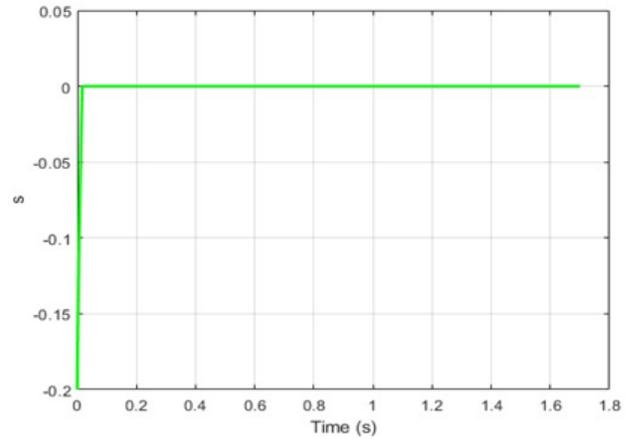


Figure 5 Sliding surface stability

Fig. 5. illustrates the convergence of the sliding surface towards zero as characterized by the classic sliding mode control mechanism. This behavior indicates the stability of the system, which is substantiated by the principles of Lyapunov stability theory. According to this theory, a system is deemed stable if the Lyapunov function is positive and its derivative demonstrates convergence towards zero over time.

The implication of this criterion is significant; it reinforces the notion that the system will consistently maintain its equilibrium state. In this context, the behavior observed within the sliding surface suggests that any deviations from the desired trajectory are progressively mitigated, leading to a stable operational regime. The successful convergence towards zero not only reflects the robustness of the control strategy employed but also highlights the effectiveness of the sliding mode in managing dynamic variables within the system. By utilizing the Lyapunov function as a tool for assessment, we can confidently assert that the stability of the system is maintained throughout its operational cycle.

This framework enhances our understanding of how advanced control methodologies can be effectively applied to optimize system performance, particularly in scenarios where maintaining stability is critical, such as in Anti-lock Braking Systems (ABS), where precise control is essential for preventing wheel lock-up during braking maneuvers.

Fig. 6. presents the error between the desired slip and the actual slip, illustrating that this error reaches zero from the moment of convergence until the completion of the braking process. This behavior is crucial in the context of Anti-lock Braking Systems (ABS), as it signifies that the system effectively eliminates any discrepancies between the intended slip ratio and the actual slip experienced by the wheels. Throughout this phase, the control mechanism ensures that the vehicle maintains optimal traction and stability, which is essential for effective braking performance. The fact that the error remains at zero indicates that the ABS is effectively responding to dynamic changes in road conditions and vehicle dynamics, thereby ensuring that the wheels do not lock up during braking.

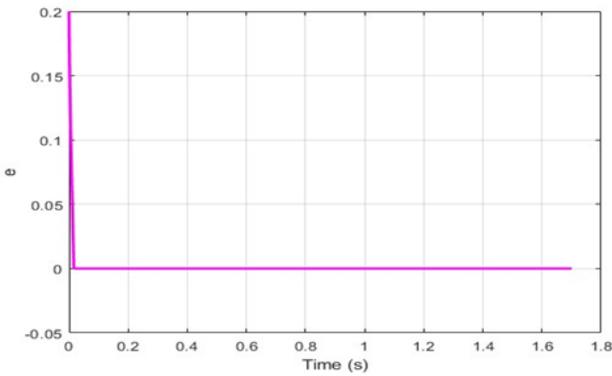


Figure 6 Response of error between the desired slip and the actual slip

Consequently, this precise regulation promotes enhanced vehicle control, preventing skidding and improving stopping distances. Overall, the ability to achieve and maintain zero error throughout the braking duration underscores the effectiveness of the control strategy employed in the ABS, which is integral to achieving safe and efficient braking behavior.

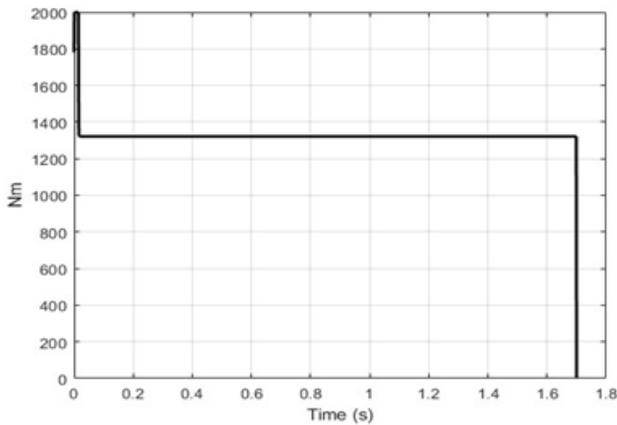


Figure 7 Braking torque behavior with the classic SMC

Fig. 7. illustrates the behavior of braking torque during the braking process, capturing the dynamics of torque response as it transitions from 1800 Nm to 2000 Nm. This range indicates an initial increase in braking force applied to the wheels, reflecting the system's active engagement to reduce vehicle speed effectively. Subsequently, the torque decreases to approximately 1500 Nm, at which point it stabilizes until the end of the braking period, which lasts for a total of 1.7 seconds.

This stabilization phase is crucial, as it signifies that the ABS is effectively managing the braking torque to optimize traction and control. The gradual shift in braking torque response demonstrates the system's ability to adjust dynamically to changing conditions, maintaining a balance between effective deceleration and preventing wheel lock-up. This precise modulation of torque contributes to an optimal longitudinal slip ratio, ensuring that the vehicle adheres to the desired slip behavior throughout the braking interval.

As the vehicle slows down, this control of braking torque is essential for maintaining stability and safety, preventing undesirable skidding or loss of control, and ultimately enhancing the overall

effectiveness of the Anti-lock Braking System. Thus, the torque behavior depicted in the figure underscores the intricate relationship between braking force, vehicle speed, and slip management in advanced vehicle dynamics

Scenario 2: With high-order ($n > 2$) fractional derivative sliding mode controller

In this second scenario with have the response behavior of the vehicle dynamic with the parameters below using the fractional derivatif sliding mode control in fine time : $k_1 = 10$; $k_2 = 10$; $k_3 = 0.01$; $K_{11} = 15$ and $\varphi_{FOFTDSMC} = 0.03$. Considering the Eq.20 in fractional order.

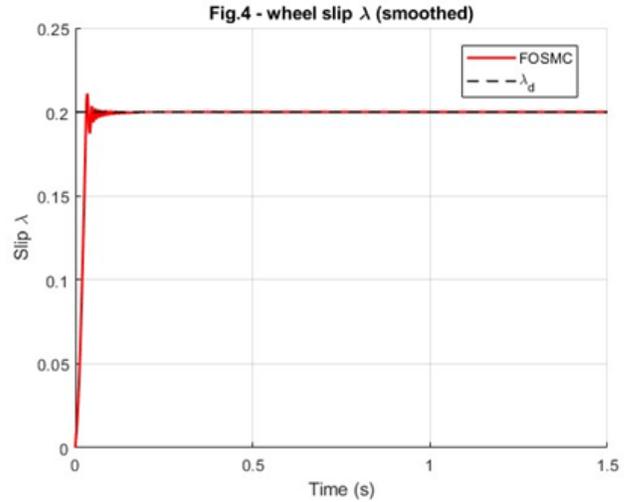


Figure 8 Longitudinal slip respons with the fractional SMC

Fig. 8. illustrates the behavior of longitudinal slip as managed by the second-order fractional derivative sliding mode control. This approach is founded on the principle that the system closely adheres to the desired reference slip $\lambda = 0.2$, thereby ensuring optimal performance during braking scenarios, $t = 1.5$ Seconds. The negligible convergence time to the reference slip signifies that the controller can rapidly adapt to variations, effectively maintaining target slip values. This swift response is crucial for enhancing vehicle stability and control during braking, particularly in emergency situations that demand quick adjustments. As shown in Fig. 4., the vehicle stops precisely within $t = 1.7$ seconds, highlighting the effectiveness of this control mechanism in achieving the desired outcome without significant delays with the fractional order.

By adhering closely to the reference slip, the system mitigates the risk of wheel lock-up, thereby maintaining traction and control during braking events. The second-order fractional derivative sliding mode control adeptly balances the need for a rapid response with stability, ensuring consistent vehicle behavior under diverse braking conditions. The findings highlight the importance of this control strategy in modern Anti-lock Braking Systems (ABS), significantly improving safety and performance.

The figure also contrasts the behaviors of classic sliding mode control, represented in blue, with fractional derivative control, depicted in green. The swift convergence time of the fractional derivative approach, showcased in red, is particularly notable when compared to the classic method, even under braking conditions utilizing fine time and higher order sliding mode techniques. Furthermore, the minimal occurrence of chattering indicates the

control's effectiveness across diverse road conditions. This reinforces the notion that fractional-order control is an optimal design choice, especially considering the system's non-linear characteristics where both robustness and rapid convergence are crucial. Ultimately, this advanced control method exemplifies its potential to further enhance braking technology, proving superior in scenarios that demand both quick responses and stability.

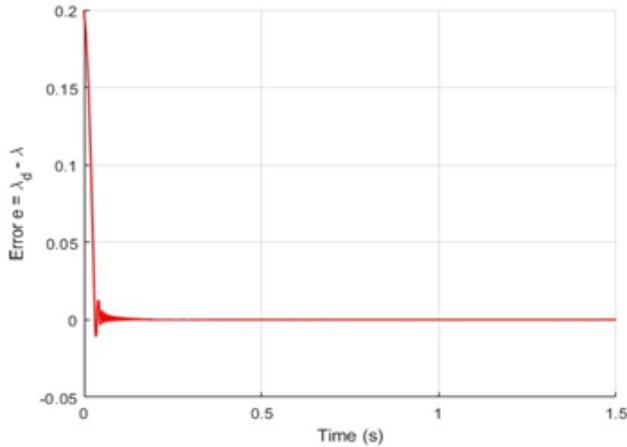


Figure 9 Error between actual and desired slip with the fractional SMC

To validate the robustness and stability of the results discussed in Fig. 8., Fig. 9. illustrates the error behavior of the second-order fractional sliding mode control. In this figure, it is evident that the control system converges to zero at the same time as the longitudinal slip response, reinforcing the effectiveness of the fractional derivative approach in maintaining stability and precision in varying conditions.

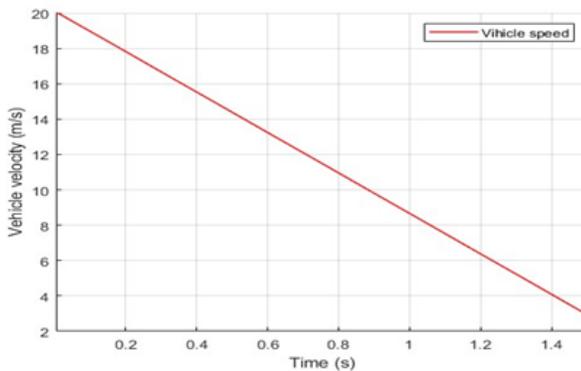


Figure 10 Vehicle velocity scenario with the fractional SMC

Figs. 10. and 11. illustrate the dynamics of the vehicle, specifically focusing on velocity and braking torque controlled by the second-order fractional sliding mode. The braking torque reflects the vehicle's ability to decelerate effectively until it comes to a complete stop, as discussed in Fig. 7. In Fig. 11., following this peak, the torque decreases to approximately 1321 Nm, at which point it stabilizes until the conclusion of the braking period, lasting a total of 1.5 seconds. This stabilization is critical, as it illustrates how the Anti-lock Braking System (ABS) effectively manages braking torque to optimize both traction and control.

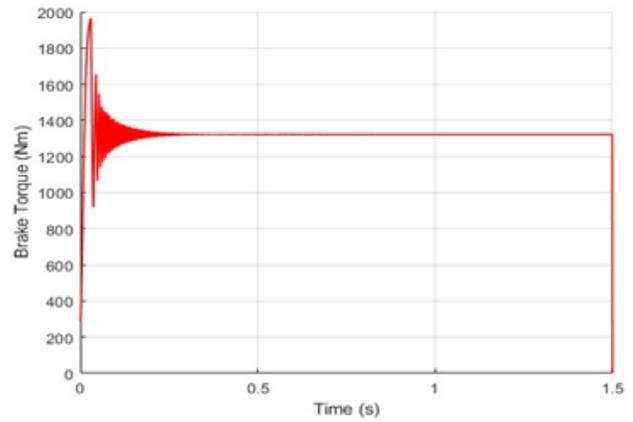


Figure 11 Braking torque behavior with the fractional SMC

The gradual adjustment in braking torque response highlights the system's capacity to dynamically adapt to varying conditions, ensuring a balance between effective deceleration and the prevention of wheel lock-up. The precise modulation of torque achieved through the fractional-order sliding mode contributes to maintaining an optimal longitudinal slip ratio, allowing the vehicle to adhere to the desired slip behavior throughout the braking process.

As the vehicle decelerates, this meticulous control of braking torque is vital for ensuring stability and safety, preventing undesirable skidding or loss of control, and ultimately enhancing the overall effectiveness of the ABS. Thus, the torque dynamics depicted in these figures emphasize the significance of employing fractional-order and high-order sliding mode control in advanced vehicle dynamics. These methodologies not only improve response times and stability but also reinforce the vehicle's performance under various braking conditions, showcasing their critical role in modern braking systems.

Influence of the fractional order derivative on the braking distance

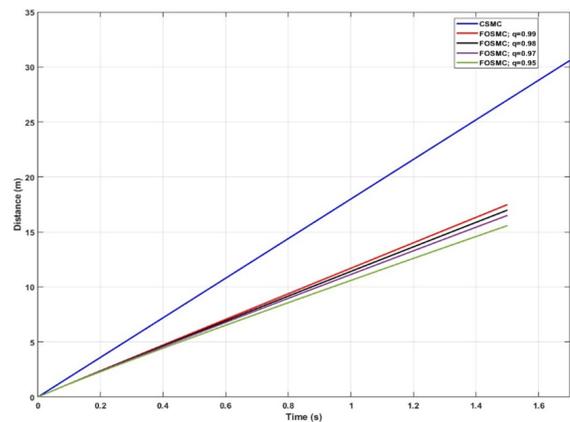


Figure 12 CSMC distance compare to the FOSMC braking distance

The influence of the fractional derivative order on the distance travelled is of fundamental importance in the design of control systems, particularly when compared to conventional sliding mode

control methods, such as those used in ABS systems. The adoption of a fractional order modifies the generally linear relationship between distance and time by introducing exponential effects, which allows the system to reach the braking distances more quickly. In terms of system responsiveness, a fractional order, such as the one shown in red in Fig. 12. with a value of 0.99 for the FOSMC, improves response speed, facilitating faster progression of distances travelled over short intervals. Conversely, the conventional sliding mode controller (CSMS), shown in blue, exhibits different performance. It can be seen that with a low derivative order value, as indicated by the green curve with an order of 0.95, the braking distance is significantly improved. Although the CSMS offers a predictable response, its lack of agility in the face of rapid changes in the control environment makes it less effective. The overall stability and performance of the system are also determined by the choice of fractional order.

Inappropriate values can cause oscillations or unpredictable behaviour, compromising control effectiveness. On the other hand, carefully selected fractional orders not only maximise dynamic performance but also ensure enhanced robustness against disturbances. Although conventional sliding mode control methods provide appreciable stability and predictability, the integration of fractional orders offers significant optimisation opportunities in terms of responsiveness and adaptability, thereby improving the performance of modern control systems.

Performance Index

Table 3 presents the performance indices for comparison purposes, specifically the root mean square error (RMSE), the integral of time-weighted absolute error (ITAE) and the integral of squared error (ITE). This table demonstrates that modeling ABS system using fractional order methods offers greater control flexibility than integer order approaches.

The ITAE is particularly significant as it emphasizes sustained errors over time, penalizing deviations that linger, which is crucial in scenarios such as emergency braking where rapid response is essential. Conversely, the ITE focuses on the overall energy of the error, assigning greater weight to larger deviations, thus ensuring that significant overshoots or undershoots are heavily penalized. In the context of ABS, where maintaining optimal slip is vital for both safety and control, lower ITAE and ITE values reflect an effective control strategy. Fractional order methods, such as fractional order sliding mode control (FOSMC), provide enhanced flexibility and a more accurate modeling of system dynamics, resulting in improved performance indices. These methods enable smoother adjustments to variations in road conditions and vehicle dynamics, leading to notable reductions in error metrics. Additionally, FOSMC demonstrates increased robustness against external disturbances, which is crucial during critical braking situations. Overall, the findings illustrate that fractional order methodologies not only outperform integer order approaches in achieving lower ITAE and ITE values but also significantly contribute to enhanced vehicle safety and performance during braking.

Fig.13. depicts the performance indices ITAE and ITE. This illustration showcases comparative results for various control strategies implemented in anti-lock braking systems. By presenting these indices, the figure emphasizes the effectiveness of fractional order methods compared to integer order methods in achieving lower error metrics. This indicates greater control flexibility and responsiveness in maintaining optimal slip during braking. The visual comparison highlights the benefits of fractional order methodologies in enhancing overall vehicle safety and performance. Ad-

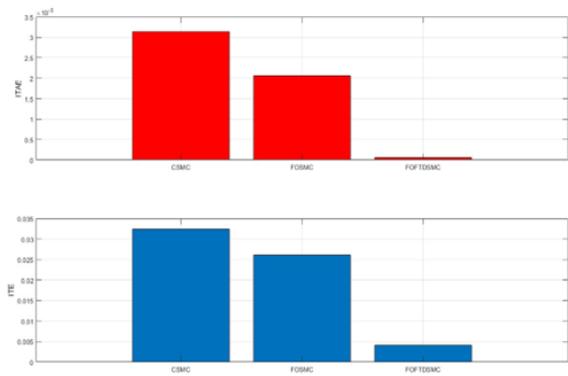


Figure 13 ITAE and ITE performance index

ditionally, selecting an appropriate degree of derivative order is critical.

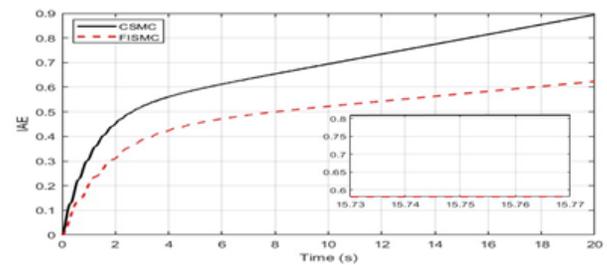


Figure 14 IAE performance index

Fig.14. represents the integral absolute error (IAE) serves as an essential performance index for comparing classic sliding mode control (CSMC) and fractional sliding mode control (FSMC). While CSMC is known for its robustness against parameter uncertainties and disturbances, it often suffers from chattering, which can result in larger absolute errors and higher IAE values, indicating promised stability and performance. In contrast, FSMC utilizes fractional calculus to achieve smoother control actions, thereby reducing chattering and enhancing overall stability. This approach allows FSMC to adapt more effectively to dynamic conditions, typically yielding lower IAE values during varying disturbances. Consequently, FSMC demonstrates superior performance in accurately tracking desired outputs, particularly in complex and nonlinear environments, showcasing its advantages over traditional SMC techniques.

CONCLUSION

This comparative analysis conclusively demonstrates the superior robustness and performance of Fractional-Order Sliding Mode Control (FOSMC) over Classic Sliding Mode Control (CSMC) for Anti-lock Braking System (ABS) design. While CSMC provides a reliable foundation for stability under nominal conditions, its rigid structure limits adaptability during rapid transients or emergency scenarios, potentially compromising braking efficacy. In contrast, FOSMC, through the incorporation of fractional derivatives, delivers a refined, adaptive control response. Key advantages established include: (a) Enhanced Convergence and Efficiency: Faster convergence to optimal slip ratios, improving braking performance

■ **Table 3** Performance Index

Controller	Stopping Time (s)	RMSE Slip	Max Control	Derivative order	ITAE	ITE
SMC	1.70	0.032	High	–	0.003140	0.032347
FOSMC	1.50	0.018	Moderate	0.99	0.000062	0.004048

during critical maneuvers. (b) Superior Dynamic Response: Reduced response times and effective mitigation of wheel lock-up risk, enhancing vehicle stability. (c) Optimized Torque Modulation: Dynamic regulation of braking torque ensures an optimal balance between deceleration and traction control. (d) Increased Robustness: Significantly reduced control chattering maintains performance across variable and adverse road conditions. The implications extend beyond ABS, suggesting FOSMC's viability for broader nonlinear control applications. This study affirms that integrating advanced strategies like FOSMC is critical for next-generation automotive safety systems. We strongly advocate for its adoption in the design of adaptive ABS controllers, as it represents a transformative advancement toward more reliable, responsive, and safer braking technology. Prioritizing such robust control solutions is essential for pioneering innovations in vehicle dynamics and enhancing overall driving safety.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

Availability of data and material

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Declaration of generative AI and AI-assisted technologies in the writing process

The authors declare that generative artificial intelligence (AI) tools were used during the preparation of this manuscript. Specifically, AI assistance was utilized for language editing, text refinement, and formatting purposes. The authors take full responsibility for the content and have carefully reviewed and verified all AI-assisted outputs.

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