

# A Study on Hindmarsh-Rose Neurons under an Electric Field

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**ABSTRACT** Today, we are at the heart of a great revolution brought about by emerging new ideas of chaos. The discovery of chaos has had a major impact on many fields of science, engineering, and mathematics. This phenomenon sheds new light on explaining the workings of the Earth's weather system, lasers, fluids, mechanical structures, earthquakes, etc. Understanding the brain and its behavior has been an active research field with various applications, including finding new solutions to cure brain diseases, designing better robots, and studying the behavior of neural networks. So far, various neural models have been developed. One such model is the Hindmarsh-Rose biological neuron model, which mimics the thalamic neurons of the brain. In this study, we analyzed the behavior of the Hindmarsh-Rose neurons under an electric field with a certain parameter. The Hindmarsh-Rose neuron model used here enables us to simulate how neurons behave in various situations, such as when they are exposed to electric fields. A program developed in MATLAB was used to perform simulations. Time response plots were obtained by varying parameters influencing the Hindmarsh-Rose neuron model. In this article, we look at how these factors change the way neurons act. Sometimes, they go from a steady firing pattern to more complex behavior, like oscillation death, which is shown by the simulations done in MATLAB software. From this article, one can understand how to improve a neural network for artificial intelligence. Additionally, how different external stimuli affect brain activity, which can lead to various neurological disorders.

**KEYWORDS**  
Hindmarsh–Rose neuron model  
Membrane potential  
Synchronization  
Quiescent  
Coupled system

## INTRODUCTION

Chaos and nonlinear dynamics have provided new theoretical and conceptual tools that allow us to capture, understand, and link together the surprisingly complex behaviors of simple systems (Hilborn 2000). The irregular and unpredictable time evolution of nonlinear systems may be termed Chaos (Baker and Gollub 1996). Chaotic behavior means erratic and almost random behavior, strongly influenced by outside noise or a system with many degrees of freedom, each doing its own thing. But these systems are deterministic. The key element in understanding this notion is nonlinearity, and the study of nonlinear behavior is nonlinear dynamics (Hilborn 2000). Many have become interested in chaos because (i) the study of chaos has provided new conceptual and theoretical tools enabling us to categorize and understand complex behavior that had confounded previous theories. (ii) Chaotic behavior seems to be universal (Hilborn 2000). The ability to transform chaotic behavior into periodic behavior would be advantageous in many day-to-day situations, such as the feasibility of using chaos control to stabilize periodic behavior from irregular heart-muscle activity (Walleczek 2000).

## CHAOS IN THE BRAIN

In this section, we will familiarize ourselves with neurons, their functions, and all the information needed to realize a neuron model, especially the Hindmarsh-Rose neuron model.

### Dynamics of a neuron

The primary building block of the central nervous system is called a neuron. The coordination between the brain and different organs is made possible with the help of these neurons through electrical impulses. Every time an input stimulus is given to a living body, the brain initiates a propagating change in the membrane potential, which brings out the response to the stimulus. This dynamic electrical excitation is called an action potential. They are characterized by sudden and transient changes of membrane potential that propagate to other neurons via a long extension called an axon.

The action potentials are created by the depolarization of the membrane (a sharp increase in the membrane potential), followed by repolarization (a slow decrease towards the resting potential). Since the action potential plays an important role in controlling the body and mind, efforts were taken to analyze its properties. The neurons are excitable because they are near a transition, called a bifurcation, from the resting phase to sustained spiking activity. Consider a neuron in its resting phase. In such a neuron, there are no changes in the membrane potential or any other state variables; hence, it is at an equilibrium point. The hyperpolarizing outward currents will balance all the inward currents, causing

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depolarization. If the neurons remain quiescent despite all the small perturbations, we can conclude that the equilibrium point is stable. In the case of a neuron, small perturbations result in small departures from the equilibrium, denoted as PSP (postsynaptic potential). The neuron's intrinsic dynamics amplify larger perturbations and result in the spike response. If we inject a sufficiently strong current into the neuron, it exhibits periodic spiking activity (Izhikevich 2007).

### Neuron model

The biological neuron model, also known as a spiking neuron model, is a mathematical description of the properties of certain cells within the systema nervosum that generate sharp electrical potentials across their cell wall (Gerstner *et al.* 2002). Neuron models are often divided into two categories consistent with the physical units of the model's interface.

Electrical input-output membrane voltage models – These models generate predictions of membrane output voltage as a function of electrical stimulation of the input stage (either voltage or current). The diverse models of this category differ from each other in the exact functional relationship between the input current and output voltage and the level of detail. This category includes models like the Hodgkin–Huxley model, FitzHugh–Nagumo model, Morris–Lecar model, and Hindmarsh–Rose model (Hodgkin and Huxley 1952).

Natural or pharmacological input neuron models – The models belonging to this category connect the input stimulus, which may be either pharmacological or natural, to the probability of a spike event. The input stage of those models isn't electrical but rather has either pharmacological (chemical) concentration units or physical units that characterize an external stimulus like light, sound, or other sorts of physical pressure. Moreover, the output stage portrays not an electrical voltage but the probability of a spike event.

### Hindmarsh-Rose neuron model

The Hindmarsh-Rose (H-R) model for neurons was developed by J. L. Hindmarsh and R. M. Rose to study the rapid firing or bursting in neurons (Coombes *et al.* 2005; Thottil and Ignatius 2019). The Hindmarsh-Rose neuron model is a simplified model of the Hodgkin-Huxley model and a modification of the FitzHugh-Nagumo model. The HR model differs in many ways from the FHN model in terms of the topology of the phase space, threshold for spikes, the way the spike trains are created, and how bursting is shut off (Mustafa *et al.* 2013). The FitzHugh-Nagumo model simplified the Hodgkin-Huxley model for neurons, but it came with many drawbacks. The FHN model reduced the complexity of the neuron models, but bursting, which is one of the most essential characteristics of neurons, could not be observed. As this model consists of only a few parameters, it was difficult to adapt this model to neurons with specific properties. They do not give a reasonable frequency–current relationship (Hindmarsh and Rose 1982). This model could explain only the generation and propagation of action potentials with only the sodium and potassium channels. It also failed to explain the rhythm of the spike train. Therefore, more channels with slower kinetics have to be introduced to better understand the underlying mechanisms. Hence, the FHN model was modified to give the Hindmarsh-Rose model, which is a three-dimensional model for a neuron with rapid firing, bursting behavior, and chaos.

The H-R neuron model is aimed at studying the spiking-bursting behavior of the membrane potential of a single neuron.

A chain of action potentials emitted by a single neuron is called a spike train; a sequence of stereotyped events that occur at regular or irregular intervals (Gerstner *et al.* 2002). The bursting behavior of the neurons, that is, the transition of a neuron from the resting phase to a recurring firing state, depends on the slow adaptation variable  $z(t)$  (Gerstner *et al.* 2002). Each burst will have a definite number of spikes unless they are in the chaotic region. In the H-R model, the relevant variable is the membrane potential  $x(t)$ . There are two more variables,  $y(t)$  and  $z(t)$ , that describe the transport of ions across membranes by ion channels. Therefore, the state of the system at any point in time is represented by the time-dependent state variables:  $x(t)$ ,  $y(t)$ , and  $z(t)$ . The Sodium and potassium ions are transported through fast ion channels, and their rate is measured by  $y(t)$ , called the spiking variable (Hindmarsh and Rose 1984). The other ions are transported through slow channels, whose rate is measured by  $z(t)$ , which is called the bursting variable (slow adaptation variable) (Hindmarsh and Rose 1984). The Hindmarsh–Rose model has the mathematical form of a system of three nonlinear ordinary differential equations with the dimensionless dynamic variables  $x(t)$ ,  $y(t)$ , and  $z(t)$  (Hindmarsh and Rose 1984). These equations are as follows:

$$\begin{cases} \dot{x} = y - ax^3 + bx^2 - z + I_{ext} \\ \dot{y} = c - dx^2 - y \\ \dot{z} = r(s(x - x_0))z \end{cases} \quad (1)$$

In these equations,  $x(t)$  represents the membrane potential, and  $y(t)$  and  $z(t)$  are recovery and adaptation variables, which account for fast and slow ion currents, respectively. 'I' represents the external stimuli or the applied current. We choose the parameters as  $a=1$ ,  $b=3$ ,  $c=1$ ,  $d=5$ ,  $r=0.006$ , and  $s=4$  so that phenomena like bursting and spiking are observed (Storace *et al.* 2008). In the Hindmarsh-Rose model, the first two equations govern spiking.

The responses of this model to a current largely depend on the values of  $r$  and  $s$ . The slow parameter ' $r$ ' controls the speed of variation in the slow variable  $z(t)$ . In the presence of spiking behavior, it governs the spiking frequency, whereas in the case of bursting, it determines the number of spikes per burst. ' $x_0$ ' is the resting potential of the system. The parameter " $s$ " allows one to switch between the bursting and spiking behavior of the neurons and thus affects the qualitative behavior of the neurons (Storace *et al.* 2008). There are three modes of operation in the full Hindmarsh-Rose model: Quiescent, Spiking, and Bursting. The quiescent mode corresponds to the absence of stable cycles. Spiking means the continuous generation of action potentials, either regular or irregular. Bursting means that action potentials arrive in clear bursts at regular or irregular periods (Lange 2006).

### Synchronization

Synchronization of chaos refers to a process where two or more chaotic systems (either equivalent or nonequivalent) adjust a given property of their motion to a common behavior due to a coupling or to a forcing (periodical or noisy) (Boccaletti *et al.* 2002). It is a complex, dynamic process and not a state. Chaotic systems are very sensitive to initial conditions and are difficult to predict. Chaotic systems with positive Lyapunov exponents resist synchronization. Two identical independent chaotic systems starting at nearly the same initial points quickly diverge with respect to time. When the Lyapunov exponents become negative, the systems start synchronizing (Boccaletti *et al.* 2002). If the trajectories of two chaotic systems are given by, say, ' $x$ ' and ' $y$ ', then they are said to be synchronized if  $x-y = 0$ .

**Coupled Systems:** Based on the way the coupling is done, unidirectional or bidirectional (mutual) coupling is possible. In the case of unidirectional coupling, a global system is formed by two sub-systems that realize a drive–response (or master-slave) configuration. This implies that one subsystem evolves freely and drives the evolution of the other (Boccaletti *et al.* 2002). Typical examples are communication with chaos. In bidirectional coupling, both subsystems are connected, and the coupling factor causes a synchronization of the rhythms between the systems. This situation typically occurs in nonlinear optics, e.g., coupled laser systems with feedback (Boccaletti *et al.* 2002). In other words, when the evolution of one of the coupled systems is unaltered by the coupling, the resulting configuration is called unidirectional coupling or drive–response coupling. Conversely, bidirectional coupling takes place when both systems are linked in a manner that results in them impacting each other’s behavior.

**Different types of synchronization:** Complete synchronization- Coupled identical systems display complete synchronization with a strong coupling strength. Here, the synchronization appears as the equality of the state variables while evolving in time. It is also known as identical or conventional synchronization (Pyragas 1996). It was first discovered and is the simplest form of synchronization in chaotic systems. This mechanism was first shown to occur when two identical chaotic systems are coupled unidirectionally, provided that the conditional Lyapunov exponents of the subsystem to be synchronized are all negative (Boccaletti *et al.* 2002).

Generalized synchronization- This type of synchronization is observed when the coupled systems are completely different. The driven (slave) and the driving (master) systems can be represented by a one-to-one mapping given by  $y(t)=x(t)$ . Thus,  $y(t)$  can be determined if the evolution of the drive system is known. Once the two systems get synchronized, the difference in trajectories, with respect to time, reduces to zero (Boccaletti *et al.* 2002).

Phase synchronization- This was first observed in the Rössler system and can be simulated by a very weak external force. If  $m$  and  $n$  are integers and  $\alpha$  and  $\beta$  represent the phases belonging to two different systems, then  $m\alpha-n\beta=C$ , where  $C$  is a constant, representing the phase synchronization relationship between the two systems. This means that phase synchronization occurs when either the phases change in the same way or a constant ratio exists between the two systems. So, perfect phase synchronization between two coupled oscillators will occur when the chaotic oscillators are phase coherent. It is achieved at very low coupling strengths (Boccaletti *et al.* 2002).

Lag synchronization- It is displayed by non-identical systems with a larger coupling strength. Lag synchronization lies between phase synchronization and complete synchronization. In lag synchronization, the states of two oscillators are nearly identical, but one system lags in time behind the other. With a slight increase in the coupling strength, complete synchronization can be achieved (Boccaletti *et al.* 2002).

Anti-synchronization- It occurs when the state variables of both the driving and driven systems are the same in magnitude but opposite in sign. In this case, the relationship between the slave,  $y(t)$ , and the master,  $x(t)$  (Boccaletti *et al.* 2002), is given by  $y(t) = -x(t)$ .

## DYNAMICS OF HINDMARSH-ROSE MODEL

### Electrical activities in the H-R neuron under an Electric field

A neuron contains many charged ions such as calcium, potassium, and sodium. Some of these ions are transported within the cell,

while others pass through the membrane channel to form trans-membrane currents. As a result, fluctuation of membrane potential occurs, and the membrane can be regarded as a charged surface with a certain uniform distribution of charges because charged ions continue to supply the flow of current (Ma *et al.* 2018).

Therefore, the surface of the cell or the membrane can be considered as a large plate with charges, and thus an electric field is induced. Suppose that the membrane holds size  $S$ , charge number  $q$ , surface charge density  $\sigma (=q/S)$ , then the intensity of the electric field close to the membrane (Ma *et al.* 2018) can be calculated by:

$$E = \frac{q}{2\epsilon_1 S} = \frac{\sigma}{2\epsilon_1} \quad (2)$$

$$\Delta V = k_1 E = E\sqrt{S} \quad (3)$$

where parameter  $\epsilon_1$  denotes the dielectric constant which is associated with the intrinsic property of the media,  $k_1$  is the radius size when the cell is regarded as a ball shape, and  $\Delta V$  represents the voltage between plates or the membrane potential of the cell. A biological neuron model should consider the effect of ion channels, which decide the propagation of ions and the membrane potential as well (Ma *et al.* 2018). However, the involvement of field variable  $E$  can well describe the distribution of ions and the change of membrane potential induced by the exchange and transport of ions in the cell (Ma *et al.* 2018).

Therefore, an electric field can be used as a new variable to estimate the change in the membrane potential of the neuron. The improved four-variable Hindmarsh–Rose neuron model can be built, including the effect of electric field as follows:

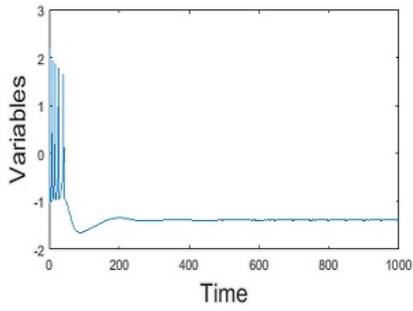
$$\begin{cases} \frac{dx_1}{dt} = x_2 - ax_1^3 + bx_1^2 - x_3 + I_{ext} \\ \frac{dx_2}{dt} = c - dx_2^2 - x_2 + k_1 E \\ \frac{dx_3}{dt} = r(s(x_1 - x_0) - x_3) \\ \frac{dE}{dt} = k_2 x_2 + E_{ext} \end{cases} \quad (4)$$

The variables  $x_1$ ,  $x_2$ , and  $x_3$  describe the membrane potential, the recovery variable for fast currents associated with potassium and sodium ions, and the adaptation current dependent on the slow current for calcium ions, respectively. Therefore, the effect of the electric field is considered by adding  $k_1 E$  to modulate the second variable  $y$ , rather than the third variable  $z$ , because the fast current is sensitive to the change of polarized and induced electric field.  $E_{ext}$  represents the external electric field, which can be a periodical modulation or noise-like radiation as well.  $k_2$  is a constant that describes the polarization, and  $I_{ext}$  is the external forcing current.

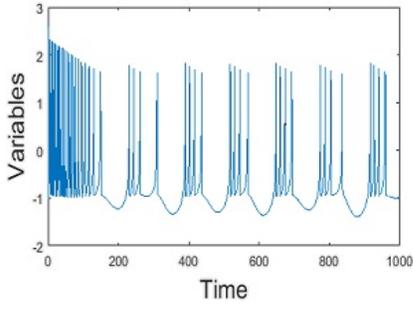
### Simulation results of the H-R neuron under an Electric field

**Time series behavior under external non-periodic current:** A variety of modes in electrical activities can be triggered by varying the external non-periodic field as  $I_{ext}=0.5\text{mA}$ ,  $3\text{mA}$ ,  $26\text{mA}$ , respectively, and the parameter values chosen as  $a=1$ ,  $b=3$ ,  $c=1$ ,  $d=5$ ,  $r=0.006$ ,  $s=4$ ,  $x_0=-1.6$ ,  $k_1=0.0001$ ,  $k_2=15$ ,  $E=0.5$ . Here, the external electric field  $E_{ext}$  is taken to be a periodical modulation, given by  $E_{ext} = B_1\sin(\omega_1 t) + B_2\cos(\omega_2 t)$ , where  $B_1$ ,  $B_2$  are the intensities while  $\omega_1$ ,  $\omega_2$  are the angular frequencies of the external electric field, respectively.

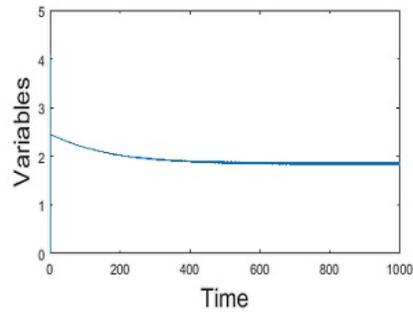
**Time series behavior under external periodic current:** Different electric modes of time series of the membrane potential of a neuron due to external periodic current are obtained by applying a periodic current  $I_{ext}=A\sin(\omega t)$ . The parameter values are selected as



(a)



(b)



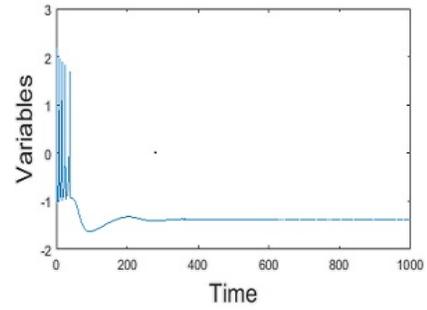
(c)

**Figure 1** Time series for membrane potential  $x_1$  are shown at different non-periodic currents (a)  $I_{ext}=0.5\text{mA}$  (b)  $I_{ext}=3\text{mA}$ , and (c)  $I_{ext}=26\text{mA}$ . The quiescent states for the spiking activities are observed at a very low external current ( $I_{ext}=0.5\text{mA}$ ). As we increase  $I_{ext}$  to  $3\text{mA}$ , we can see the periodic bursting behavior of the neuron. The system settles down to oscillation suppression only after applying a very high external current.

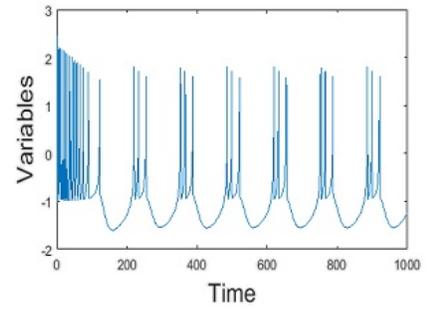
$a=1, b=3, c=1, d=5, r=0.006, s=4, x_0=-1.6, k_1=0.0001, k_2=15, E=0.5, B_1=50, B_2=0.5, \omega_1=5.5, \omega_2=3.5$  and  $t=1$ . Here, the different modes are obtained by varying the amplitude of the current while the angular frequency is fixed.

#### Electrical activities in a Synchronized H-R neuron under an Electric field and noise

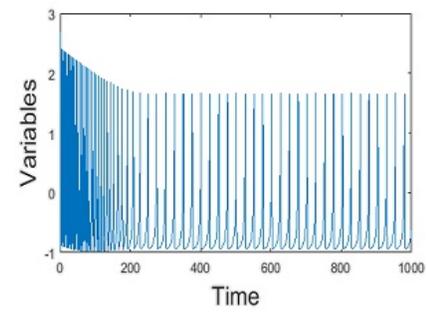
$$\begin{cases} \frac{dx_1}{dt} = x_2 - ax_1^3 + bx_1^2 - x_3 + I_{ext} + g(x_5 - x_1) \\ \frac{dx_2}{dt} = c - dx_1^2 - x_2 + k_1E \\ \frac{dx_3}{dt} = r(s(x_1 - x_0) - x_3) \\ \frac{dE}{dt} = k_2x_2 + E_{ext} \end{cases} \quad (5)$$



(a)



(b)



(c)

**Figure 2** Influence of periodic current on membrane potential is shown in figure for (a)  $A=7, \omega=0.15$ , (b)  $A=15, \omega=0.15$ , and (c)  $A=25, \omega=0.15$ , respectively. From the figure, the quiescent states in the first figure change to quiescent states with sustained bursting along with an increase in the periodic current ( $A=15, \omega=0.15$ ). The quiescent states change to tonic spiking as the amplitude of the external periodic current is very high ( $A=25, \omega=0.15$ ).

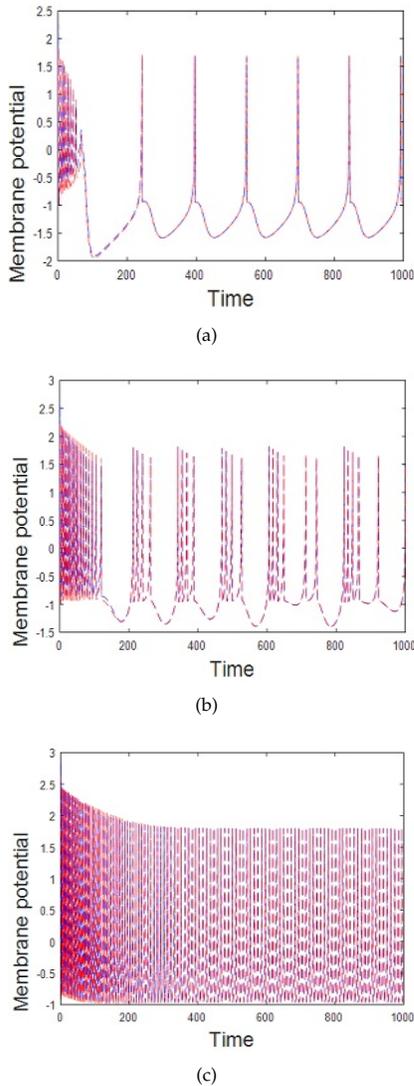
$$\begin{cases} \frac{dx_4}{dt} = x_6 - ax_5^3 + bx_5^2 - x_7 + I_{ext} + g(x_1 - x_5) \\ \frac{dx_5}{dt} = c - dx_5^2 - x_6 + k_1E \\ \frac{dx_6}{dt} = r(s(x_5 - x_0) - x_7) \\ \frac{dE}{dt} = k_2x_6 + E_{ext} \end{cases} \quad (6)$$

Where  $x_1, x_2$ , and  $x_3$  represent the membrane potential, slow current, and adaptation current of the first neuron, respectively. Similarly,  $x_4, x_5$ , and  $x_6$  represent the corresponding variables of the second neuron.  $I_{ext}$  is the external forcing current. The parameter values are selected as  $a=1, b=3, c=1, d=5, r=0.006, s=4, x_0=-1.6$ . The term  $g$  represents the coupling intensity between the neurons (Pyragas 1996).  $k_1$  is the radius size,  $E_{ext}$  is the external

electrical field, and  $k_2$  describes the polarization.

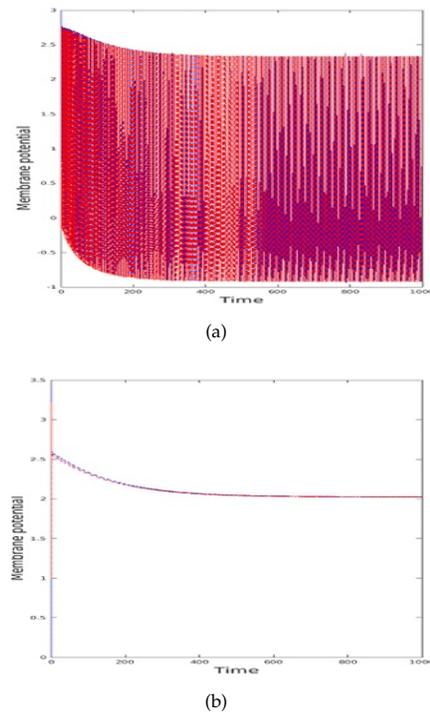
### Simulation results of the Synchronized H-R neuron under an Electric field and noise

**Time series behavior under the Electric field:** Time series for membrane potentials for the two coupled neurons for different external forcing currents, (a)  $I_{ext}=1.5\text{mA}$ , (b)  $I_{ext}=3\text{mA}$ , and (c)  $I_{ext}=5\text{mA}$ .



**Figure 3** Here the parameter values are selected as  $k_1=0.0001$ ,  $k_2=15$ ,  $E=0.5$ ,  $B_1=50$ ,  $B_2=0.5$ ,  $\omega_1=5.5$ ,  $\omega_2=3.5$ ,  $t=1$  and  $g=1$ . When  $I_{ext}=1.5\text{mA}$ , the system exhibits a periodic behavior. Also, when the external forcing current is set as  $I_{ext}=3\text{mA}$ , chaotic synchronization is observed. A transition from chaotic to tonic synchronization states is observed when the external periodic current increases to  $I_{ext}=5\text{mA}$ .

**Time series behavior under the noise:** Similarly, we applied an external electric field,  $E_{ext}=\zeta(t)$  as a noise-like disturbance, where  $\zeta(t)$  is the Gaussian white noise. By keeping the external forcing current fixed, say  $I_{ext}=5\text{mA}$ , and changing the intensity of noise as (a)  $D=5$  and (b)  $D=25$ , different states of the neurons are observed.



**Figure 4** Time series for membrane potentials are plotted for fixed external forcing current ( $I_{ext}=5\text{mA}$ ) for different noise parameters (a)  $g=1, D=5$  and (b)  $g=1, D=25$ . At  $I_{ext}=5\text{mA}$ , the system shows a tonic behavior even if the external electric field is either a periodic modulation or noise-like radiation. But as we increase the noise intensity to a very high value, say  $D=25$ , the system changes its state from tonic to that of oscillation death.

### CONCLUSION

The human brain is made up of billions of neurons that communicate with each other through chemical and electrical synapses. Neurons exhibit varying firing patterns, including spiking, bursting, chaotic firing, and a combination of these patterns. Once two neurons are coupled, their dynamics become complex and display different forms of synchronization, amplitude death, oscillation death, and near-death-like spikes (Resmi *et al.* 2011). Here, we studied the Hindmarsh-Rose (H-R) model under an electric field. A detailed study of the Hindmarsh-Rose model for neurons is done with the help of MATLAB software. In the first part, we analyzed the different modes of electrical activities of the membrane potential in a single neuron under the external electric field term. The system behavior is studied under the influence of external periodic and non-periodic currents. It is observed that for a non-periodic current, as the value of the external current increases, the quiescent states become broadened, and for higher values of external current, the system settles down to an oscillation death state. Nevertheless, in the case of the periodic current, the action potential shows enhanced quiescent states for the spiking activities. In the higher current, the neuron exhibits tonic oscillations in contrast to the suppression of activities observed in the non-periodic case.

The focal point of the study also includes the synchronization of coupled neurons. Under the influence of the electric field, the system changes through periodic-, chaotic-, and tonic-type synchronization as the current is increased. Also, from the introduction of control parameters like noise on the neurons, it is found

that when the noise factor is applied to the system, the oscillation death is achieved for a smaller magnitude of external current. This highlights a crucial aspect of neurophysiological resilience and flexibility. The ability of the electric field generated by the brain to influence its activity appears to be particularly notable during epileptic seizures. However, the effect of an electric field is not limited to these pathological conditions. The study by Frohlich and McCormick demonstrates that the electrical fields also influence brain function during normal activities such as sleep. Studies based on biological models also have high potential for developing clinical treatment modes for diseases such as Alzheimer's. The detailed study of the dynamical behaviors of single and coupled neurons helps in comprehending higher functions of the brain, like perception, episodic memory, learning, awareness, etc. The observations made in the study might point to new means of curing disorders resulting from irregular neuronal synchronization, such as epilepsy and other neurodegenerative conditions. Future work may explore the influence of electric fields on synaptic behavior. One may also investigate the effects of these influences on different neural circuits that control higher cognitive functions. An understanding of these interactions may lead to new methods to enhance cognitive abilities and address many of the neurological disorders, in addition to the more effective linkage of theoretical models to practical applications in neuroscience.

#### Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

#### Availability of data and material

Not applicable.

#### Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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